Logarithms in economics: an example

Suppose that a firm's output Y is related to capital input K and labour input L by the production function

$$Y = \tfrac{1}{5}K^{0.4}L^{0.6}. \qquad (4.10)$$

Taking logarithms (all to the same base) and applying **L1** and **L2**,

$$\log Y = -\log 5 + \log\left(K^{0.4}\right) + \log\left(L^{0.6}\right).$$

Let $y = \log Y$, $k = \log K$ and $\ell = \log L$. Then by **L4**,

$$y = 0.4k + 0.6\ell - \log 5. \qquad (4.11)$$

Equation (4.11) is a linear relationship between the logarithms of the economic variables. For this reason, relations such as (4.10) are said to be **log-linear**. The procedure of transforming a non-linear relationship into a linear one by taking logarithms is a very useful one, and we shall return to it in Chapter 9.

Exercises

4.4.1 Evaluate, without using a calculator, the following:

(a) $\log_5 125$, (b) $\log_5(1/125)$, (c) $\log_8 64$, (d) $\log_8 4$, (e) $\log_8 256$.

4.4.2 Use the properties of logarithms to show that $\log_a x = 1/\log_x a$.

4.4.3 A firm's output Y is related to capital input K, labour input L and natural resource input R by the production function

$$Y = 2K^{1/2}L^{1/3}R^{1/6}.$$

Write down a linear relationship between the logarithms to base 10 of Y, K, L, R.

Problems on Chapter 4

4–1. Solve simultaneously the equations $q = 5 - p^2$, $q = 2p - 3$.
Now suppose p represents price and q quantity, and that the equations represent demand and supply curves respectively. Sketch the parts of the curves which lie in the non-negative quadrant; state the equilibrium price and quantity.

4–2. This problem is concerned with a class of functions which are not quadratic but can be minimised by completing the square.

(i) Let the function $f(x)$ be defined for $x > 0$ by

$$f(x) = ax + b + c/x,$$

where a, b, c are constants such that $a > 0$ and $c > 0$. By expressing $f(x)$ in the form

$$\left(\sqrt{ax} - \sqrt{c/x}\right)^2 + \text{constant},$$

find the value of x that minimises $f(x)$. Hence find the minimum value of $f(x)$.

(ii) The cost function of a firm is

$$C(x) = 50 + 2x + 0.08x^2,$$

where x is output. Using the result of (i), find the level of output that minimises average cost $C(x)/x$. Also find the minimum average cost.

4–3. A firm's output Y is related to capital input K and labour input L by the production function

$$Y = 2K^{2/3}L^{1/3}.$$

(i) Suppose that initially $K = a$ and $L = b$. Find the percentage increase in Y resulting from 1% increases in both K and L. What happens when the changes are each 10%? Can you formulate a general result?

(ii) Assuming that $K = 27$, draw a diagram showing how Y is related to L. Also draw a diagram showing how $\log_{10} Y$ is related to $\log_{10} L$.

(iii) Assuming that $Y = 20$, draw a diagram showing how K is related to L. Also draw a diagram showing how $\log_{10} K$ is related to $\log_{10} L$.

4–4. Suppose the supply and demand functions for petrol are respectively

$$q = \tfrac{1}{4}p^4, \quad q = 8p^{-1},$$

where p represents price and q represents quantity.
Sketch a standard supply-and-demand diagram for petrol (q on the horizontal axis, p on the vertical) and find the equilibrium price and quantity.
Also find the equilibrium price and quantity by another method: express the supply and demand functions as relations between the logarithms of the price and quantity, and solve the resulting pair of simultaneous linear equations.