5.4.3 In each of the following cases, state whether there are real numbers a and b such that the function $y = f(x)$ is continuous, and find them if they exist.

$$\text{(a) } f(x) = \begin{cases} ax^3 & \text{if } x \le 2, \\ bx^2 & \text{if } x > 2. \end{cases} \qquad \text{(b) } f(x) = \begin{cases} x + a^2 & \text{if } x \le 2, \\ x - b^2 & \text{if } x > 2. \end{cases}$$

$$\text{(c) } f(x) = \begin{cases} x^3 + a^2 & \text{if } x \le 2, \\ x^2 - b^2 & \text{if } x > 2. \end{cases}$$

5.4.4 Verify that the equation $x^5 + 3x = 12$ has a solution between 1 and 2. Do you think that there is a solution (a) between 1 and $\frac{3}{2}$, (b) between $\frac{3}{2}$ and 2?

Problems on Chapter 5

5–1. Show that the sequence

$$u_n = \frac{3n}{n+3}$$

tends to a limit z. Find z. For this sequence, make a table of values giving $p = 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}$ and a corresponding integer N that satisfies (5.1).

Does the sequence

$$v_n = \frac{3n^2}{n^3+3}$$

tend to a limit?

Do either of the functions

$$y = \frac{3x}{x+3}, \qquad y = \frac{3x^2}{x^3+3}$$