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(iii) For each real number
$$b \neq 0$$
, the function g_b is defined as follows:
$$g_b(x) = \frac{x^b - 1}{b} \quad (x \geq 0)$$
:

Also let $g_0(x) = \ln x$. Use the second result of (i) to show that $\lim_{x \to 0} g_0(x) = g_0(x) \quad \text{for all } x > 0,$

Find $g_h^{\mu}(x)$, $g_h^{\mu}(x)$, $g_h(1)$ and $g_h(1)$. Use these results to sketch the eurose Find $g_h^{\mu}(x)$, $g_h^{\mu}(x)$, $g_h(1)$ and $g_h(1)$. In the same xy-plane for $b=0,\frac{1}{2},\pm 1$ and ± 2 .

 $y = g_b(x)$ in the same xy-plane for $b = 0, \frac{1}{2}, \pm 1$ and $\pm 2, \pm 1$ $y = g_k(x)$ in the same xy-plante y. The functions of part (ii) are known in financial economics as 'constant absolute risk are the coefficient of absolute risk are [The functions of part (ii) are known in many the coefficient of absolute risk aversion risk aversion (CARA) utility functions; the coefficient of absolute risk aversion risk aversion (CARA) utility functions; the coefficient of absolute risk aversion (CARA) utility functions; the coefficient of absolute risk aversion (CARA) utility functions; the coefficient of absolute risk aversion (CARA) utility functions; the coefficient of absolute risk aversion risk aversion (CARA) utility functions; the coefficient of absolute risk aversion risk aversion (CARA) utility functions; the coefficient of absolute risk aversion risk aversion (CARA) utility functions; the coefficient of absolute risk aversion risk aversion (CARA) utility functions. risk aversion' (CARA) utility functions: the risk aversion' (CARA) utility functions in statistics as 'Box-Cox transforms is -a. The functions of part (iii) are known in statistics as 'Box-Cox transforms is -a. The functions of part (iii) are known in statistics as 'Box-Cox transforms is -a. is -a. The functions of part (iii) are known relative risk aversion' (CRRA) trility tions' and in financial economics as 'constant relative risk aversion is 1-b.) functions: the coefficient of relative risk aversion is $1=b_{i}$]

(i) As in part (i) of Problem 5–4, a timber owner plants a forest at time 0. The

As in part (i) of Problem 3—1, a time $t \ge 0$ is given by the function f(t), volume of timber in the forest at time $t \ge 0$ is given by the function f(t), volume of timber in the lorest at the solution of timber in the lorest at the price per unit volume of wood harvested is p and the interest rate is r, the price per unit volume of wood harvested is p and the interest rate is r. The price per unit volume of the constant, p is net of harvesting costs and Both p and r are assumed to be constant, p is net of harvesting costs and planting costs are ignored.

Suppose now that time is measured continuously and discounting is consuppose now that r being the instantaneous rate of interest. If the forest is cut down and the timber sold at time T, what is the value of the forest at time 0? Show that this value, considered as a function of T, has a critical point where T satisfies

$$f'(T)/f(T) = r. (9.10)$$

[Equation (9.10) is called the Fisher rule, after the famous American economist Irving Fisher (1867-1947).]

(ii) Assume everything is as in (i), except that at time $T, 2T, 3T, \ldots$ the timber owner cuts down the forest, sells the timber and replants it with similar trees. Express the value of the forest at time 0 as a function of T, and show that it has a critical point where

$$f'(T)/f(T) = re^{rT}/(e^{rT} - 1)$$
 (9.10')

[This equation is called the Faustmann rule, after the nineteenth-century German landowner and forester Martin Faustmann.]

- (iii) Show that the right-hand side of (9.10') approaches 1/T as $r \to 0$. [This means that, when the interest rate is very low, the Faustmann rule approximates the 'forester's rule' of Problem 8-2.]
- (iv) Assuming f(t) has the shape specified in Problem 8–2, discuss the circumstances in which the station stances in which the stationary points of parts (i) and (ii) are global maxima.

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It is large, the express

But by (9.1), the limit respectively. E2 now f

Differentiating the e

We now give an expl

A few minutes compound interess APR is very small, h is small then th (exph-1-h)/h

 $^{^{7}}T$ is called the **rotation period**, and does not have to be an integer.